INTERNATIONAL INDIAN SCHOOL, RIYADH

FIRST - TERM WORKSHEET

CHAPTER 3 – MATRICES

1. Let A = $\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$, then show that $A^2 - 4A + 7I = 0$. Using the above, calculate A^5 also.
2. Find non zero values of x satisfying
   $x\begin{bmatrix} 2x & 2 \\ 3 & x \end{bmatrix} + 2\begin{bmatrix} 8 & 5x \\ 4 & 4x \end{bmatrix} = 2\begin{bmatrix} x^2+8 & 24 \\ 10 & 6x \end{bmatrix}$
3. If A is a square matrix X such that $A^2 = A$ show that $(I + A)^3 = 7A + I$
4. Find the matrix X such that
   $\begin{bmatrix} 2 & -1 \\ 0 & 1 \\ -2 & 4 \end{bmatrix} X = \begin{bmatrix} -1 & -8 & -10 \\ 3 & 4 & 0 \\ 10 & 20 & 10 \end{bmatrix}$
5. If A = $\begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ and B = $\begin{bmatrix} -2 & -1 & -4 \end{bmatrix}$
   Verify that $(AB)^{-1} = B^1A^1$
6. If A and B are symmetrical, then show that (AB + BA) is symmetric and (AB – BA) is skew symmetric.
7. If $f(x) = x^2 - 4x + 1$, find f(A) given A = $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$
8. Find x if $\begin{bmatrix} x & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x \\ 3 \end{bmatrix} = 0$
9. Find the inverse by elementary transformations of

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

10. If $A = \begin{bmatrix} 1 & 3 & 5 \\ -2 & 5 & 7 \end{bmatrix}$ and $2A - 3B = \begin{bmatrix} 4 & 5 & -9 \\ 1 & 2 & 3 \end{bmatrix}$

find B.

11. If $A = \begin{bmatrix} Cos \propto & Sin \propto \\ -Sin \propto & Cos \propto \end{bmatrix}$, show that

$$A^2 = \begin{bmatrix} Cos\ 2 \propto & Sin\ 2 \propto \\ -Sin\ 2 \propto & Cos\ 2 \propto \end{bmatrix}$$

12. Given $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$, find a and b such that $A^2 + aI = bA$
13. If $A = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$, find p and q so that $(pI + qA)^2 = A$
14. If $A = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, find $(A + 2B)^1$
15. If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, prove that $(aI + bA)^3 = a^3I + 3a^2bA$
16. Show that all the elements of the main diagonal of a skew symmetric matrix are zero.
17. Express as the sum of a symmetric and a skew symmetric matrix and verify, given $A = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$
18. If $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$, find x + y
19. Using elementary operations, find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$$

20. If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, show that $(aI + bA)^n = a^nI + na^{n-1}bA$ using mathematical induction.